# Useful Math Overview

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CSC 212

Based on slides from Prof. Ned Okie, Radford University
https://www.radford.edu/~nokie/classes/360/math.html

# Announcements

- Piazza and Gradescope accounts setup
- Class is full! No more new students can be admitted unless someone drops
- IoT Sunday – Makerfest.
- Quiz0 on Thursday (Sept 12) --- will cover today's material --- NO CALCULATORS

# Introduction

- We will frequently use basic math in this course
  - You need to be familiar with it
- Topics include
  - Exponentiation
  - Logarithms
  - Permutation and Combination
  - Summations
  - Floor and ceiling
  - Factorials

# Exponentiation

- Addition, multiplication, exponentiation:
  - **Multiplication** is repeated addition
  - **Exponentiation** is repeated multiplication

- Example:
  - 2 * 3 = 2 + 2 + 2
  - $2^{3} = 2 * 2 * 2$

- General idea
  - **Addition:** $a + n = a + (1 + \cdots + 1)$ *(n times)*
  - **Multiplication:** $a \times n = a + a + \cdots + a$ ($n$ times)
  - **Exponentiation:** $a\hat{}n = a \times a \times \cdots \times a$ ($n$ times)

# Properties of Powers

- Properties follow from the definition:
  - $b^a * b^c = ?$
  - $b^0 = ?$
  - $b^a/b^c = ?$
  - $1 / b^c = ?$
  - $(b^a)^c = ?$

Examples:

  - $2^2 * 2^{30} = ?$
  - $2^{10} / 2^4 = ?$
  - $2^{-3} = ?$
  - $(2^3)^4 =?$

# Important Approximations

- $2^{10} = 1024 \approx 1000 = 10^3$
- $2^{20} = (2^{10})^2 = 1024^2 \approx 1000^2 = (10^3)^{2} = 10^6 = 1$ million
- $2^{30} \approx$ ??
- $2^{32} \approx$ ??
- $2^{64} \approx$ ??

# Logarithms

- *Definition*
  - $log_b x$ is defined as the **power** to which the base, *b*, must be raised to produce *x*
- So, $b^{log_b x} = ?$
  - (assuming $b > 0$ and $x > 0$)

# Logarithm Intuition

- Another way to think about logarithms.

- **Definition:** $\log_b x$ *i*s the **power to which we raise b to get *x***

- **Intuition:** $\log_b x$ is **the number of times *x* can be divided by *b* before reaching 1**

- **Logarithm can be calculated for any positive base**

# Examples

- $\log_8 64 = ?$
- $\log_5 625 = ?$
- $\log_7 16807 = ?$
- $\log_e 20 = ?$

**$\log_2$ = lg (log to the base two)**

**$\log_{10}$ = log (log to the base 10)**

- $\log_{15} 3375 = ?$

**$\log_e$ = ln (natural logarithm)**
$e \approx 2.71828$

- $\log_{10}$ 1trillion = ?

[12 zeros]

# Important Properties of Logarithms

- $\log_b -1$ = ?, for any *b*
- $\log_b 1$ = ?, for any *b*
- $\log_b b$ = ?, for any *b*
- $\log_b (b^x)$ = ?     for any b
- $b^{\log_b x} = ?$
- $\log_b xy$ = ?
- $\log_b x^y$ = *?*
- $\log_b x/y$ ?

# More Examples

- $2^{\lg 7} = ?$
- $\lg (4 * 8) = ?$
- $\lg(1/4) = ?$
- $\lg (16 / 2) = ?$
- $log_{3^2} 729 = ?$
- $\lg (16^5) = ?$

# Approximations

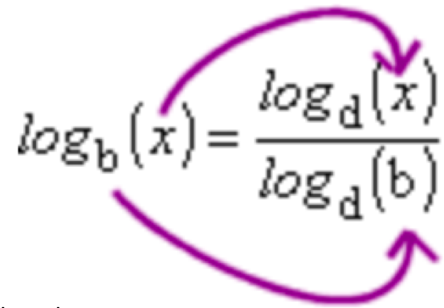
- What if the power is not an integer?
  - $\log_2 4 = 2.000$
  - $\log_2 7 \approx 2.807$
  - $\log_2 8 = 3.000$

- Intuition:
  - If *x* is an integer power of *b*, it's exactly the number of times to divide, otherwise approximately

# Changing Logarithm Base

**Change-of-Base Formula:**

$$\log_b(x) = \frac{\log_d(x)}{\log_d(b)}$$




Courtesy: https://www.purplemath.com/modules/logrules5.htm

$\log_4 35 = \log(35) / \log(4) = 1.544/0.6 = 2.57..$

$\log_4 35 = \lg(35) / \lg(4) = 5.13/2 = 2.57...$

Derivation can be found here: https://www.algebra.com/algebra/homework/logarithm/gonzo-lesson-1.lesson

# Calculate using Change-of-Base Rule

- $\log_{9}81 = ?$
- $\log_{7.38}e = ?$
- $\log_{17}1024 = ?$
- $\log_{103}4096 = ?$
- $\log_{1024}65536 = ?$
- $\log_{49}343 = ?$

# Factorials

- Definition: For a positive integer n>= 0
  - n! = 1, if n = 0
  - n! = n * (n-1)! If n > 0

- Thus n!  = 1*2*3*....*n

- 5! = 1*2*3*4*5 = 120
- 10! = 3628800
- 20! = 2.4 * $10^{18}$

**Grows Exponentially**

# Permutation

- Definition: ordering of a set of objects
- In practice: how many ways can you order *n* objects?
- Example --- permutations of abc
  - abc, acb, bac, bca, cab, cba
- Number of permutation of n objects: $\boldsymbol{P(n)=n!}$
  - WHY?

# Combination

- c(*n*,*k*) is the number of ways of choosing k objects from n objects when order does not matter
  - The text describes *c*(*n*,*k*) as the number of *k*-combinations of an *n*-element set
  - Also written as (*nk*)
  - Spoken as "*n* choose *k*"
- $c(n,k) = \dfrac{n!}{(k!(n-k)!)} = \dfrac{n*(n-1)*(n-2)*...*(n-k+1)}{k!}$

# Basic Summation Formulas

$$\sum_{i=1}^{n} c = c + c + c + \ldots + c \text{ (n times)} = cn, \text{ where } c \text{ is a constant.}$$

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \; .$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \; .$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4} \; .$$

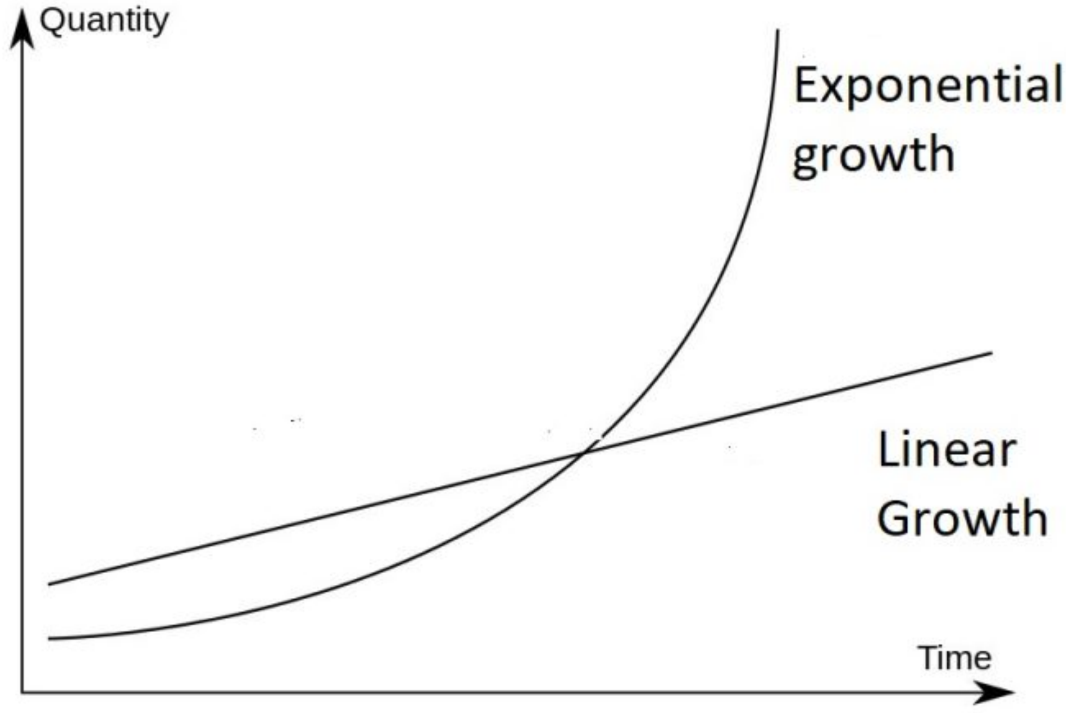
Courtesy: https://www.math.ucdavis.edu/~kouba/CalcTwoDIRECTORY/summationdirectory/Summation.html

# Floor and Ceiling

- Floor(*x*) = the largest integer that is ≤ *x* floor(*x*) is written as $\lfloor x \rfloor$
  - On a number line, floor(*x*) is the first integer at or **left** of *x* (ie towards −∞) for positive *x*
  - Floor(*x*) truncates the fractional part of *x*
  - Example $\lfloor 2.5 \rfloor$ = ???

- Ceiling(*x*) = the smallest integer that is ≥ x ceiling(*x*) is written as $\lceil x \rceil$
  - On a number line, ceiling (*x*) is the first integer at or **right** of *x* (ie toward ∞) for negative *x*
  - Ceiling (*x*) truncates the fractional part of *x*
  - Example $\lceil 4.2963884 \rceil$ = ???

# Linear Growth vs Exponential Growth



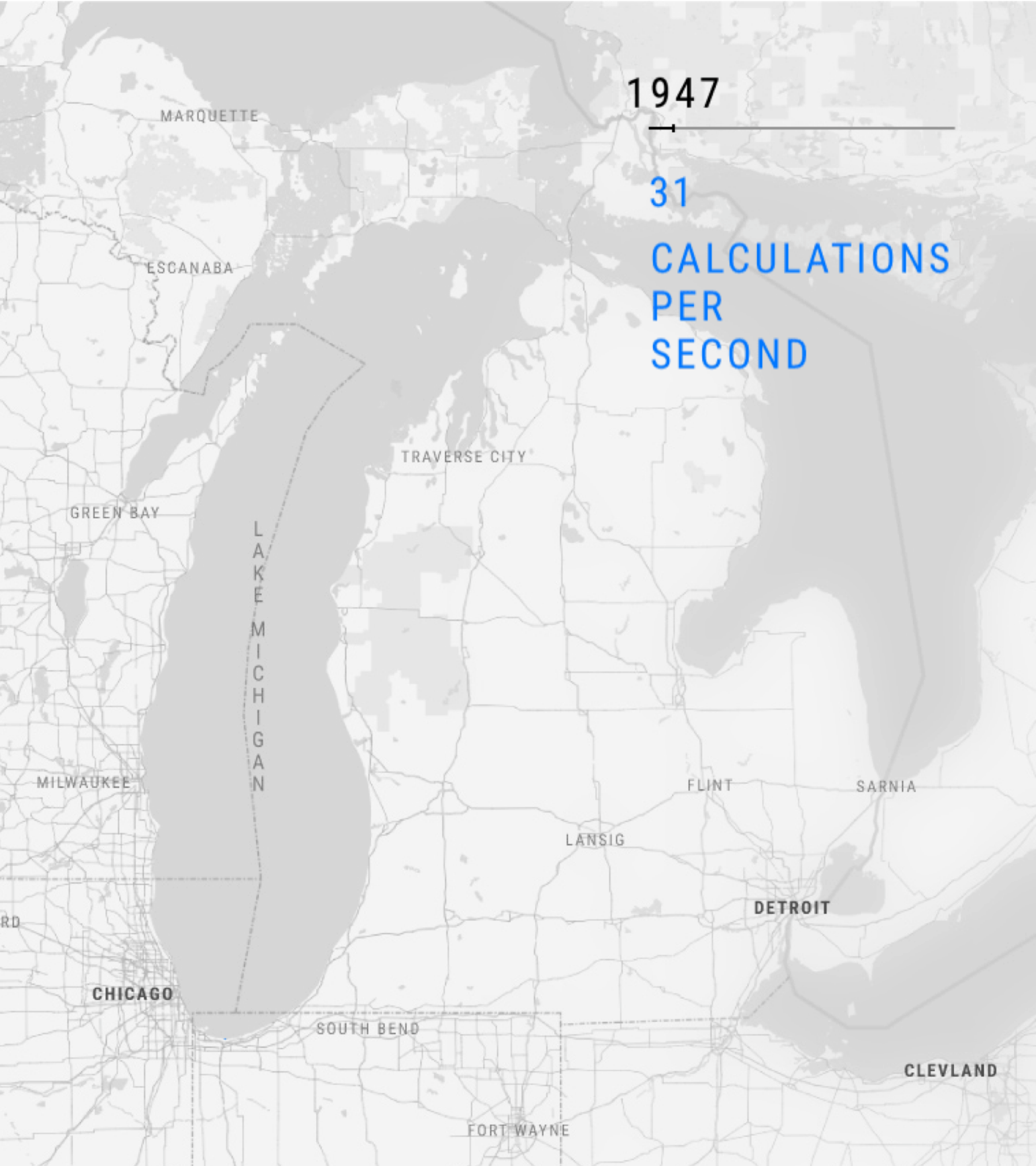
## How Long Until Computers Have the Same Power As the Human Brain?

Lake Michigan's volume (at $2.88 \times 10^{17}$ fluid ounces) is about the same as our brain's capacity (in calculations per second). According to Moore's Law, computer processing power doubles every 18 months. If you want to fill Lake Michigan with water at that rate, you will see very little progress for a long time, and then suddenly it fills all the way up.

1947

31

CALCULATIONS PER SECOND

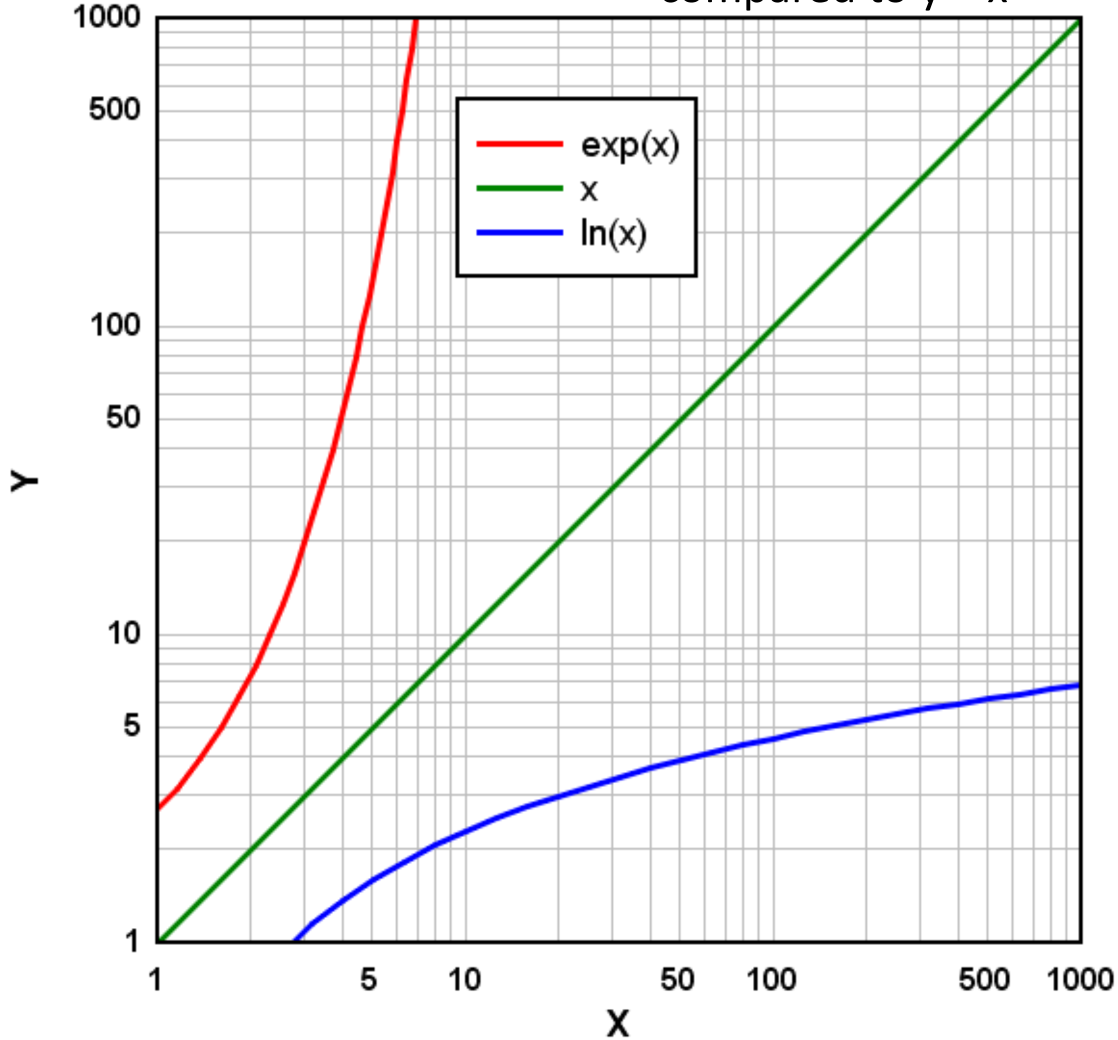



200 KM OR 124 MILES

Source: https://singularityhub.com/2016/04/05/how-to-think-exponentially-and-better-predict-the-future/

# Comparison

Example y = $2^x$ grows **exponentially** compared to y = x




y = x grows **exponentially** compared to y = ln x

Or for any log function

# Practice

- Solve these.

  - Show $\log_b a = 1/ \log_a b$
  - Simplify $log_{b^n}$ a
  - Show $b^{\log_a d} = d^{\log_a b}$
  - If *log (x+1)- log(x-1) = 2*, what is *x*?
  - What is $\sum_{i=1}^{n} i - 1$ ?
  - Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?
  - From a group of 7 women and 6 men, five persons are to be selected to form a committee so that at least 3 women are there on the committee. In how many ways can it be done?


That's all Folks!
Any Question?